Localization with the Extended Kalman Filter

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*Abstract*—This paper details the implementation of an EKF localization filter for the case study of a bot traversing a 10x10 m square on an open field.

1. Introduction

A bot equipped with a lidar sensor and an IMU was made to traverse a 10x10 meter square path *located* at the center of an open field. A pylon was placed at the center of the square, which serves as a known location for EKF localization to be performed. An image of the experimental setup is shown in Figure 1. Data was sampled at 10 Hz.



Fig. 1. Image of data collection with bot and pylon

1. Motion Model
2. Vectors

A 5 degree of freedom state vector in the global coordinate frame was defined as shown in Equation 1. Here, and are position and velocity in the East (+) and West (-) direction. Elements and are position and velocity in the North (+) and South (-) direction. is positive counterclockwise and 0 towards East.

(1)

The control input is defined as shown in Equation 2. Here, is the lateral acceleration or in the local coordinate frame from IMU, is the forward acceleration or in the local coordinate frame from IMU, and is the yaw velocity of the robot. IMU measurement is positive clockwise.

(2)

1. Kinematics

We know that there is no lateral slip. Therefore, the acceleration in the y direction ay’,t is 0. The x and y position are defined by Equations 3-4.

(3)

(4)

The velocities vx and vy are defined by Equations 5-6. Note that the global y velocity still depends on the forward x’ acceleration not lateral y’ acceleration.

(6)

Once Equations 5-6 are plugged into Equations 3-4, we get Equations 7-8.

The yaw angle of the bot is given by Equation 9.

1. Linearizing the Motion Model

The state transition probability function can be approximated as linear with added Gaussian noise, as shown in Equation 10-11.

(10)

(11)

The Gx matrix can be calculated by taking derivatives of the kinematics equations with respect to each element in the state vector. The resulting matrix is shown in Equation 12.

(12)

The Gu matrix is calculated by taking derivatives of the kinematics equations with respect to the control inputs. There are only two columns, however, because there is no lateral slip, so a\_y is 0. The resulting matrix is shown in Equation 13.

(13)

1. Measurement vector

The measurement vector, which contains data obtained from the lidar is shown in Equation 14. The equations are referenced to (5, -5) which is the location of the pylon. Here, y\_lidar is positive in the local forward direction, and x\_lidar is positive in the local left direction.

The measurement vector only affects the first, second, and last elements of the state vector. Thus, the H matrix is created to isolate just those elements of the state vector, as shown in Equation 15.

(15)

1. Extended Kalman Filter

The Kalman filter algorithm takes in the inputs

* 1. Prediction step
  2. Correction step
  3. Sensor description and covariance matrix

1. Results
   1. Estimated path

The Kalman filter was implemented for all lidar data. The initial covariance matrix was given to be an identity matrix. The estimated path that was outputted by the filter is shown in Figure 2.

Chart

Description automatically generated

Fig. 2. Plot with estimated path from EKF

* 1. Tracking error

The tracking error of the estimated path was found by finding the shortest distance between the path and the perfect square at each point in the estimated path. The error is plotted as a function of time in Figure 3. The root mean squared error was found to 0.227.

Chart, histogram

Description automatically generated

Fig. 3. Tracking error of estimated path

* 1. Yaw plot
  2. Covariance ellipses

The confidence ellipse was plotted at multiple points on the estimated path using the 2x2 xy portion of the covariance matrix as shown in Figure 5.

* 1. Covariance matrix elements as a function of time

References

1. S. M. Metev and V. P. Veiko, *Laser Assisted Microtechnology*, 2nd ed., R. M. Osgood, Jr., Ed. Berlin, Germany: Springer-Verlag, 1998.